SADLER UNIT 4. CHAPTER 8

EXERCISE 8A

Q1 $xy + 8x = 10 - 2y$

$x\frac{dy}{dx} + y + 8 = -2\frac{dy}{dx}$

$(x+2)\frac{dy}{dx} = -y - 8$

$\frac{dy}{dx} = \frac{-y-8}{x+2}$

Q2. $xy + y - 4x = 3x^2 - 5$

$x\frac{dy}{dx} + y + \frac{dy}{dx} - 4 = 6x$

$(x+1)\frac{dy}{dx} = 6x + 4 - y$

$\frac{dy}{dx} = \frac{6x+4-y}{x+1}$

Q3. $y^3 - 2x = 3x^2 y$

$3y^2\frac{dy}{dx} - 2 = 3x^2\frac{dy}{dx} + 6xy$

$(3y^2 - 3x^2)\frac{dy}{dx} = 6xy + 2$

$\frac{dy}{dx} = \frac{6xy+2}{3y^2-3x^2}$

Q4. $y^2 = 2x^3 y + 5x$

$2y\frac{dy}{dx} = 2x^3\frac{dy}{dx} + 6x^2 y + 5$

$(2y - 2x^3)\frac{dy}{dx} = 6x^2 y + 5$

$\frac{dy}{dx} = \frac{6x^2y+5}{2y-2x^3}$

Q5. $5y^2 = x^2 + 2xy - 3x$

$10y\frac{dy}{dx} = 2x + 2x\frac{dy}{dx} + 2y - 3$

$(10y - 2x)\frac{dy}{dx} = 2x + 2y - 3$

$\frac{dy}{dx} = \frac{2x+2y-3}{10y-2x}$ //

Q6. $x + 3y^2 = 5 + x^2 + 2xy$

$1 + 6y^2\frac{dy}{dx} = 2x + 2x\frac{dy}{dx} + 2y$

$(6y^2 - 2x)\frac{dy}{dx} = 2x + 2y - 1$

$\frac{dy}{dx} = \frac{2x+2y-1}{6y^2-2x}$

Q7. $x^2 + y^2 = 9x$

$2x + 2y\frac{dy}{dx} = 9$

$2y\frac{dy}{dx} = 9 - 2x$

$\frac{dy}{dx} = \frac{9-2x}{2y}$

Q8 $x^2 + y^2 = 9y$

$2x + 2y\frac{dy}{dx} = 9\frac{dy}{dx}$

$(2y - 9)\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{2y-9}$

Q9. $x^2 + y^2 = 9xy$

$2x + 2y\frac{dy}{dx} = 9x\frac{dy}{dx} + 9y$

$(2y - 9x)\frac{dy}{dx} = 9y - 2x$

$\frac{dy}{dx} = \frac{9y-2x}{2y-9x}$

Q10. $x^2 + y^2 = 9xy + x + y$

$2x + 2y\frac{dy}{dx} = 9x\frac{dy}{dx} + 9y + 1 + \frac{dy}{dx}$

$(2y - 9x - 1)\frac{dy}{dx} = 9y + 1 - 2x$

$\frac{dy}{dx} = \frac{9y+1-2x}{2y-9x-1}$

Q11. $\sin x + \cos y = 10$

$\cos x - \sin y \frac{dy}{dx} = 0$

$-\sin y \frac{dy}{dx} = -\cos x$

$\frac{dy}{dx} = \frac{\cos x}{\sin y}$.

Q12. $3 + x^2 \cos y = 10xy$

$-x^2 \sin y \frac{dy}{dx} + 2x\cos y = 10x\frac{dy}{dx} + 10y$

$(-x^2\sin y - 10x)\frac{dy}{dx} = 10y - 2x\cos y$

$\frac{dy}{dx} = \frac{2x\cos y - 10y}{x^2\sin y + 10x}$

Q13. $6x + xy + 2a + 2y = 0$

$6 + x\frac{dy}{dx} + y + 2\frac{dy}{dx} = 0$

$(x+2)\frac{dy}{dx} = -y - 6$

$\frac{dy}{dx} = \frac{-y-6}{x+2}$

$\left.\frac{dy}{dx}\right|_{(x,y)=(-3,2)} = \frac{-8}{-1}$

$= 8$

Q14. $6y + xy = 10 + 3x$

$6\frac{dy}{dx} + x\frac{dy}{dx} + y = 3$

$(6+x)\frac{dy}{dx} = 3 - y$

$\frac{dy}{dx} = \frac{3-y}{6+x}$

$\left.\frac{dy}{dx}\right|_{(x,y)=(2,2)} = \frac{1}{8}$

Q15. $5 + x^3 = xy + y^2$

$3x^2 = x\frac{dy}{dx} + y + 2y\frac{dy}{dx}$

$3x^2 - y = (x + 2y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{3x^2 - y}{(x+2y)}$

$\left.\frac{dy}{dx}\right|_{(x,y)=(1,-3)} = \frac{3+3}{1-6}$

$= -\frac{6}{5}$

Q16. $y^2 + 3xy = 4x$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 4$

$(2y + 3x)\frac{dy}{dx} = 4 - 3y$

$\frac{dy}{dx} = \frac{4 - 3y}{(2y + 3x)}$

$\left.\frac{dy}{dx}\right|_{(x,y)=(1,-4)} = \frac{4 - 3(-4)}{-8 + 3}$

$= \frac{16}{-5}$

Q17. $x^2 + \frac{y}{x} = 2y$

$2x + \frac{x\frac{dy}{dx} - y}{x^2} = 2\frac{dy}{dx}$

sub (1,1).

$2 + \frac{dy}{dx} - 1 = 2\frac{dy}{dx}$

$1 = \frac{dy}{dx}$

$\therefore y = x + c$

$1 = 1 + c \Rightarrow c = 0$

$\therefore y = x$

Q18 $5x^2 + \sqrt{xy} = 5 + y^2$

$10x + \frac{1}{2}(xy)^{-\frac{1}{2}} \cdot \left(x\frac{dy}{dx} + y\right) = 2y\frac{dy}{dx}$

$10x + \frac{x\frac{dy}{dx} + y}{2\sqrt{xy}} = 2y\frac{dy}{dx}$

$40 + \frac{4\frac{dy}{dx} + 9}{2(6)} = 18\frac{dy}{dx}$

$480 + 4\frac{dy}{dx} + 9 = 216\frac{dy}{dx}$

$489 = 212\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{489}{212}$

Q19. $\frac{dy}{dx} = x^2y$

$\frac{d^2y}{dx^2} = x^2\frac{dy}{dx} + 2xy$

$= x^2(x^2y) + 2xy$

$= x^4y + 2xy$

Q20. $x^2 + 4y^2 - 2x + 6y = 17$

$2x + 8y\frac{dy}{dx} - 2 + 6\frac{dy}{dx} = 0$

$(8y+6)\frac{dy}{dx} = 2 - 2x$

$\frac{dy}{dx} = \frac{2-2x}{8y+6}$

$\therefore$ When $\frac{dy}{dx} = 0$, $2 - 2x = 0$

$2x = 2$

$x = 1$

$\therefore 1 + 4y^2 - 2 + 6y = 17$

$4y^2 + 6y - 18 = 0$

$2y^2 + 3y - 9 = 0$

+	3
x	-18

$2y^2 - 3y + 6y - 9 = 0$

$\Rightarrow -3, 6 \quad y(2y-3) + 3(2y-3) = 0$

$(2y-3)(y+3) = 0$

$\therefore y = \frac{3}{2}$ or $y = -3$

$\therefore \underline{(1, \frac{3}{2})}$ and $\underline{(1, -3)}$

Q21 $x^2 + y^2 - 4x + 6y + 12 = 0$

$2x + 2y\frac{dy}{dx} - 4 + 6\frac{dy}{dx} = 0$

$(2y+6)\frac{dy}{dx} = 4 - 2x$

$\frac{dy}{dx} = \frac{4-2x}{2y+6}$

vertical when $2y + 6 = 0$

$y = -3$

$\therefore x^2 + 9 - 4x - 18 + 12 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or $x = 1$

$\therefore \underline{(3, -3)}$ and $\underline{(1, -3)}$

Q22. $y - y^3 = x^2 + x - 2$

$\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 2x + 1$

$(1 - 3y^2)\frac{dy}{dx} = 2x + 1$

$\frac{dy}{dx} = \frac{2x+1}{1-3y^2}$

$\left.\frac{dy}{dx}\right|_{\substack{x=1 \\ y=0}} = \frac{3}{1} = 3$ //

$\frac{d^2y}{dx^2} - \left(3y^2\frac{d^2y}{dx^2} + 6y\frac{dy}{dx}\frac{dy}{dx}\right) = 2$

$(1-3y^2)\frac{d^2y}{dx^2} = 2 + 6y\left(\frac{2x+1}{1-3y^2}\right)^2$

$$\frac{d^2y}{dx^2} = \frac{2(1-3y^2)^2 + 6y(2x+1)^2}{(1-3y^2)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\substack{x=1\\y=0}} = \frac{2(1)^2 + 0}{1}$$

$$= 2$$

Q23. $x^2 = 2\sin y$

$$2x = 2\cos y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{2\cos y}$$

$$= \frac{x}{\cos y}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=\frac{\pi}{6}}} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{2}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$\therefore y = \frac{2\sqrt{3}}{3}x + c$$

$$\frac{\pi}{6} = \frac{2\sqrt{3}}{3} + c$$

$$c = \frac{\pi - 4\sqrt{3}}{6}$$

$$\therefore y = \frac{2\sqrt{3}}{3}x + \frac{\pi - 4\sqrt{3}}{6}$$

Q24. $y^2 + \cos x = 3y + 1$

$$2y\frac{dy}{dx} - \sin x = 3\frac{dy}{dx}$$

$$(2y-3)\frac{dy}{dx} = \sin x$$

$$\frac{dy}{dx} = \frac{\sin x}{2y-3}$$

$$2y\frac{dy}{dx} - \sin x = 3\frac{dy}{dx}$$

$$2y\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\cdot\frac{dy}{dx} - \cos x = 3\frac{d^2y}{dx^2}$$

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{\sin^2 x}{(2y-3)^2}\right) - \cos x = 3\frac{d^2y}{dx^2}$$

$$(2y-3)\frac{d^2y}{dx^2} = \frac{\cos x(2y-3)^2 - 2\sin^2 x}{(2y-3)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\cos x(2y-3)^2 - 2\sin^2 x}{(2y-3)^3}$$

Q25. $2\sin y - x^2 = 2x + 1$

$$2\cos y\frac{dy}{dx} - 2x = 2$$

$$\frac{dy}{dx} = \frac{2+2x}{2\cos y}$$

$$= \frac{1+x}{\cos y}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=\frac{\pi}{6}}} = \frac{-1}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{2\sqrt{3}}{3}$$

$$2\cos y\frac{dy}{dx} - 2x = 2$$

$$2\cos y\frac{d^2y}{dx^2} - 2\sin y\left(\frac{dy}{dx}\right)^2 - 2 = 0$$

$$2\cos y\frac{d^2y}{dx^2} = 2 + 2\sin y\left(\frac{1+x}{\cos y}\right)^2$$

$$= \frac{2\cos^2 y + 2\sin y(1+x)^2}{\cos^2 y}$$

$$\frac{d^2y}{dx^2} = \frac{2\cos^2 y + 2\sin y(1+x)^2}{2\cos^3 y}$$

$$= \frac{\cos^2 y + \sin y(1+x)^2}{\cos^3 y}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\substack{x=-2\\y=\frac{\pi}{6}}} = \frac{\frac{3}{4} + \frac{1}{2}(1)}{\frac{3\sqrt{3}}{8}} = \frac{10\sqrt{3}}{9}$$

Q26. $3x^2 + y^2 = 9$

$6x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{6x}{2y}$

$-1 = -\frac{3x}{y}$

$\boxed{y = 3x}$

$\therefore 3x^2 + 9x^2 = 9$

$12x^2 = 9$

$x^2 = \frac{9}{12}$

$x = \pm\frac{\sqrt{3}}{2}$

$\therefore y = \pm\frac{3\sqrt{3}}{2}$

$\therefore \left(\pm\frac{\sqrt{3}}{2}, \pm\frac{3\sqrt{3}}{2}\right)$

EXERCISE 8B

Q1. $x = 3\sin 2t$

$y = 2\cos 5t$

a) $\frac{dx}{dt} = 6\cos(2t)$

b) $\frac{dy}{dt} = -10\sin(5t)$

c) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{-10\sin(5t)}{6\cos(2t)}$

$= -\frac{5\sin(5t)}{3\cos(2t)}$

Q2. $x = \sin^2 t$

$y = \cos(3t)$

a) $\frac{dx}{dt} = 2\sin t\cos t$

b) $\frac{dy}{dt} = -3\sin(3t)$

c) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{-3\sin(3t)}{\sin(2t)}$

Q3. $x = 2 + 3t$

$y = t^2$

$\frac{dx}{dt} = 3 \quad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2t}{3}$

Q4. $x = t^2$

$y = 2 + 3t$

$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = 3$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{3}{2t}$

Q5. $x = 5t^3$

$y = t^2 + 2t$

$\frac{dx}{dt} = 15t^2 \quad \frac{dy}{dt} = 2t + 2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2t+2}{15t^2}$

Q6. $x = 3t^2 + 6t$

$y = \frac{1}{t+1}$

$\frac{dx}{dt} = 6t + 6$

$\frac{dy}{dt} = \frac{(t+1)(0) - 1(1)}{(t+1)^2}$

$= -\frac{1}{(t+1)^2}$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{-1}{(t+1)^2} \cdot \frac{1}{6(t+1)}$

$= -\frac{1}{6(t+1)^3}$

Q7. $x = t^2 - 1$

$y = (t-1)^2$

$\frac{dx}{dt} = 2t$

$\frac{dy}{dt} = 2(t-1)$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2(t-1)}{2t}$

$= \frac{t-1}{t}$

Q8. $x = \frac{t}{t-1}$

$y = \frac{2}{t+1}$

$\frac{dx}{dt} = \frac{(t-1)(1) - 1(t)}{(t-1)^2}$

$= \frac{t-1-t}{(t-1)^2}$

$= \frac{-1}{(t-1)^2}$

$\frac{dy}{dt} = \frac{-2}{(t+1)^2}$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{-2}{(t+1)^2} \times \frac{(t-1)^2}{-1}$

$= \frac{2(t-1)^2}{(t+1)^2}$

Q9. $x = t^2 + 2$

$y = t^3$

$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{3t^2}{2t}$

$= \frac{3t}{2}$

$\left.\frac{dy}{dx}\right|_{t=-1} = -\frac{3}{2}$

Q10. $x = \frac{1}{t+1}$

$y = t^2 + 1$

$\frac{dx}{dt} = \frac{-1}{(t+1)^2}$

$\frac{dy}{dt} = 2t$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -2t(t+1)^2$

$\left.\frac{dy}{dx}\right|_{t=2} = -4(9)$

$= -36$

Q11. $x = 2t^2 + 3t$

$y = t^3 - 12t$

$\frac{dx}{dt} = 4t + 3$

$\frac{dy}{dt} = 3t^2 - 12$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{3t^2 - 12}{4t + 3}$

$0 = 3t^2 - 12$

$12 = 3t^2$

$t^2 = 4$

$t = \pm 2$

When $t = 2$,

$x = 2(4) + 6$

$x = 14$

$y = 8 - 24$

$y = -16$

$\therefore (14, -16)$

When $t = -2$,

$x = 2(4) - 6$

$x = 2$

$y = -8 + 24$

$y = 16$

$\therefore (2, 16)$

Q12. $x = 4\sin t$

$y = 2\sin 2t$

a) $\frac{dx}{dt} = 4\cos t$

$\frac{dy}{dt} = 4\cos(2t)$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{4\cos(2t)}{4\cos(t)}$

$= \frac{\cos(2t)}{\cos t}$ //

b) $x = 4\left(\frac{1}{2}\right) = 2$

$y = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$\therefore (x, y) = (2, \sqrt{3})$

$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = \frac{\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{1}{\sqrt{3}}$

c) $0 = \frac{\cos(2t)}{\cos t}$

$\cos(2t) = 0, \quad 0 \le 2t \le 4\pi$

$2t = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\right\}$

$\therefore t = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

Q13. $y = t + \frac{2}{t}$

a) $x = 2t - \frac{1}{t}$

$\frac{dy}{dt} = 1 - \frac{2}{t^2}$

$\frac{dx}{dt} = 2 + \frac{1}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{1 - \frac{2}{t^2}}{2 + \frac{1}{t^2}}$

$= \frac{t^2 - 2}{2t^2 + 1}$

b) See next page!

$$\frac{dy}{dx} = \frac{t^2-2}{2t^2+1}$$

$$\frac{d^2y}{dx^2} = \frac{(2t^2+1)(2t\frac{dt}{dx}) - (4t\frac{dt}{dx})(t^2-2)}{(2t^2+1)^2}$$

$$= \frac{(2t(2t^2+1) - 4t(t^2-2))\frac{dt}{dx}}{(2t^2+1)^2}$$

$$= \frac{(4t^3+2t-4t^3+8t)\left(\frac{t^2}{2t^2+1}\right)}{(2t^2+1)^2}$$

$$= \frac{10t(t^2)}{(2t^2+1)^3}$$

$$= \frac{10t^3}{(2t^2+1)^3}$$

Questions 4 – 8 will be completed using implicit differentiation.

Q4. $T = \frac{2\pi}{3}\sqrt{L}$

a) $\frac{dT}{dt} = \frac{2\pi}{3}\left(\frac{1}{2}\right)L^{-\frac{1}{2}} \cdot \frac{dL}{dt}$

$= \frac{2\pi}{6\sqrt{L}} \cdot \frac{dL}{dt}$

$= \frac{2\pi}{60} \cdot \frac{15}{\pi}$

$= \frac{1}{2}$

b) $\frac{dT}{dt} = \frac{2\pi}{6\sqrt{L}}\frac{dL}{dt}$

$6\pi = \frac{\pi}{3(2)} \cdot \frac{dL}{dt}$

$36 = \frac{dL}{dt}$

EXERCISE 8C

Q1. $y = 3x^2 + 4x$

$\frac{dx}{dt} = 5$

$\frac{dy}{dx} = 6x + 4$

$\therefore \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= (6x+4)(5)$

$= 30x + 20$

$\left.\frac{dy}{dt}\right|_{x=6} = 200$

Q2 $A = 8p^3$

$\frac{dp}{dt} = 0.25$

$\frac{dA}{dp} = 24p^2$

$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$

$\frac{dA}{dt} = 24p^2\left(\frac{1}{4}\right)$

$= 6p^2$

$\left.\frac{dA}{dt}\right|_{p=\frac{1}{2}} = 6\left(\frac{1}{4}\right)$

$= \frac{3}{2}$

Q3. $x = \sin 2p$

$\frac{dp}{dt} = 2$

$\frac{dx}{dp} = 2\cos(2p)$

$\frac{dx}{dt} = \frac{dx}{dp} \times \frac{dp}{dt}$

$= 2\cos(2p)(2)$

$= 4\cos(2p)$

$\left.\frac{dx}{dt}\right|_{p=\frac{\pi}{6}} = 4\cos\left(\frac{\pi}{3}\right)$

$= 2$

Q5. $A = \sin^2(3x)$

$\frac{dA}{dt} = 2\sin(3x)3\cos(3x)\frac{dx}{dt}$

$= 6\sin(3x)\cos(3x) \cdot \frac{1}{10}$

$= \frac{3\sin 6x}{10}$

$= \frac{3}{10}\sin\left(6\left(\frac{\pi}{36}\right)\right)$

$= \frac{3}{20}$

Q6. $P = 4r^2 + 3$

$\frac{dP}{dt} = 8r\frac{dr}{dt}$

$14 = 8(7)\frac{dr}{dt}$

$\frac{14}{56} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4}$

Q7. $y^2 = 3x^3 + 1$

$2y\frac{dy}{dt} = 9x^2\frac{dx}{dt}$

$10\frac{dy}{dt} = 9x^2\frac{1}{10}$

$\frac{dy}{dt} = \frac{9x^2}{100}$

when $y = 5$,

$25 = 3x^3 + 1$

$24 = 3x^3$

$8 = x^3$

$x = 2 \Rightarrow x^2 = 4$

$\therefore \frac{dy}{dt} = \frac{36}{100}$

$= \frac{9}{25}$

Q8 $x^2 + y^2 = 400$, $x \geq 0$.

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2x(6) + 24\frac{dy}{dt} = 0$

$24\frac{dy}{dt} = -12x$

$\frac{dy}{dt} = -\frac{x}{2}$

When $y = 12$,

$x^2 + 144 = 400$

$x^2 = 256$

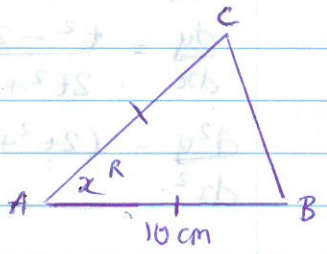
$x = 16$, $x \geq 0$

$\therefore \frac{dy}{dt} = -8$

Q9. Let A: Area

Need $\frac{dA}{dt}\Big|_{x=\frac{\pi}{3}}$

Know $\frac{dx}{dt} = 0.01$



$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(10)(10)\sin x$

$A = 50\sin x$

$\frac{dA}{dt} = 50\cos x \frac{dx}{dt}$

$= 50\left(\frac{1}{2}\right)\cdot\frac{1}{100}$

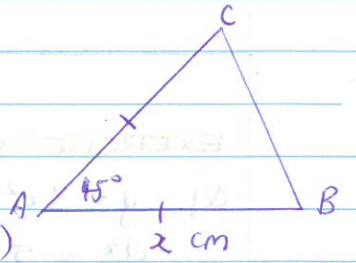
$= \frac{1}{4}$.

$\therefore$ 0.25 cm^2/sec increase

Q10. Let A: Area

Need $\frac{dA}{dt}\Big|_{x=10}$

Know $\frac{dx}{dt} = 0.1$



$A = \frac{1}{2}x^2\sin(45°)$

$A = \frac{\sqrt{2}}{4}x^2$

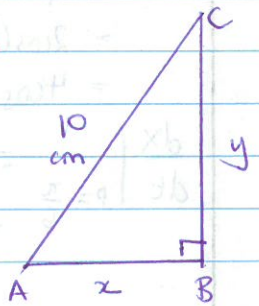
$\frac{dA}{dt} = \frac{2\sqrt{2}}{4}x\frac{dx}{dt}$

$= \frac{\sqrt{2}}{2}(10)\frac{1}{10}$

$= \frac{\sqrt{2}}{2}$.

$\therefore$ Increase of $\frac{\sqrt{2}}{2}$ cm^2/sec

Q11.



Need $\frac{dy}{dt}\Big|_{t=20}$.

Know $\frac{dx}{dt} = 0.1$

$x^2 + y^2 = 100$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

When $t = 20$, $\frac{dx}{dt} = 0.1$

$\Rightarrow x = 4 + 2.$

$\therefore 36 + y^2 = 100$

$y^2 = 64$

$\therefore y = \pm 8.$

Reject (-ve)

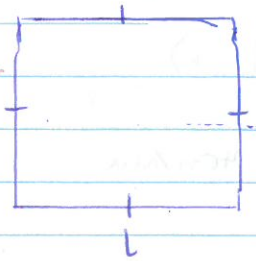
$\therefore \frac{dy}{dt} = -\frac{6.}{8}\left(\frac{1}{10}\right)$

$= -\frac{3}{40}$

$= -0.075$

$\therefore$ decrease of 0.075 cm/sec

Q12.



$\frac{dL}{dt} = 0.01$

Need $\left.\frac{dA}{dt}\right|_{L=8}$

$A = L^2$

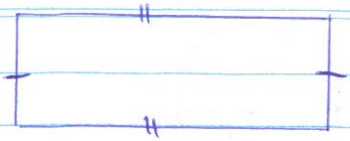
$\frac{dA}{dt} = 2L\frac{dL}{dt}$

$\left.\frac{dA}{dt}\right|_{L=8} = 16(0.01)$

$= 0.16$

$\therefore$ increases by $0.16\ cm^2/sec$

Q13



$\frac{dw}{dt} = 1$

$L = 3w$

Need $\left.\frac{dA}{dt}\right|_{w=100\ mm}$

$A = Lw$

$= 3w(w)$

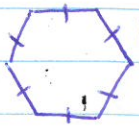
$A = 3w^2$

$\frac{dA}{dt} = 6w\frac{dw}{dt}$

$\left.\frac{dA}{dt}\right|_{w=10} = 6(100)$

$= 600\ mm^2/sec$

Q14.

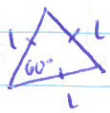


$\frac{dL}{dt} = 1\ cm/min$

Need $\left.\frac{dA}{dt}\right|_{L=20.}$

$A = 6\left(\frac{1}{2}ab\sin C^\circ\right)$

Consider



$\Rightarrow$ equilateral

$A = 6\left(\frac{1}{2}L^2\sin 60^\circ\right)$

$= 3L^2\frac{\sqrt{3}}{2}$

$A = \frac{3\sqrt{3}}{2}L^2$

$\frac{dA}{dt} = 3\sqrt{3}\,L\frac{dL}{dt}$

$\left.\frac{dA}{dt}\right|_{L=20} = 60\sqrt{3}\,(1)$

$= 60\sqrt{3}\ cm^2/min$

Q15

$V = \frac{4}{3}\pi r^3$, $\frac{dr}{dt} = 0.1$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{2}{5}\pi r^2$

a) $\left.\frac{dV}{dt}\right|_{r=5} = \frac{2(25)}{5}\pi$

$= 10\pi \text{ cm}^3/\text{s}$.

b) $40\pi = \frac{2\pi}{5} r^2$

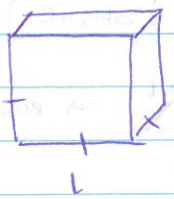
$100 = r^2$

$r = \pm 10$

(ignore −)

$\therefore r = 10$ cm

Q16.



$\frac{dl}{dt} = 0.1$

a) Need $\left.\frac{dSA}{dt}\right|_{l=10}$

$SA = 6l^2$

$\frac{dSA}{dt} = 12l \frac{dl}{dt}$

$\left.\frac{dSA}{dt}\right|_{l=10} = 120\left(\frac{1}{10}\right)$

$= 12 \text{ cm}^2/\text{sec}$

b) Need $\left.\frac{dV}{dt}\right|_{l=10}$

$V = l^3$

$\frac{dV}{dt} = 3l^2 \frac{dl}{dt}$

$\left.\frac{dV}{dt}\right|_{l=10} = 3(100)\left(\frac{1}{10}\right)$

$= 30 \text{ cm}^3/\text{sec}$

Q17.

$\frac{dV}{dt} = 5 \text{m}^3/\text{min}$ $\quad V = \pi r^2 h$

$V = 0.05\pi r^2$

a) Need $\left.\frac{dr}{dt}\right|_{r=20}$

$\frac{dV}{dt} = 0.1\pi r \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{\pi r}{10} \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{10}{\pi r} \frac{dV}{dt}$

$\left.\frac{dr}{dt}\right|_{r=20} = \frac{1}{2\pi}(5)$

$= 0.7958 \text{ m/min}$

$\approx 80 \text{ cm/min}$

b) $\frac{dr}{dt} = \frac{10}{\pi r} \frac{dV}{dt}$

$= \frac{1}{4\pi}(5)$

$= 0.3979$

$\approx 40 \text{ cm/min}$

c) $\frac{dr}{dt} = \frac{10}{\pi r} \frac{dV}{dt}$

$= \frac{1}{10\pi}(5)$

$= \frac{1}{2\pi}$

$= 0.1592$

$\approx 0.16 \text{ cm/min}$.

Q18 Know : $h = 5r$

$\frac{dr}{dt} = \frac{2}{\pi}$ mm/sec

$= \frac{2}{10\pi}$ cm/sec

a) $V = \pi r^2 h$

$= \pi r^2 (5r)$

$V = 5\pi r^3$

$\frac{dV}{dt} = 15\pi r^2 \frac{dr}{dt}$

$= \frac{15\pi r^2 (2)}{10\pi}$

$\frac{dV}{dt} = 3r^2$ cm^3/sec

b) $SA = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r (5r)$

$= 2\pi r^2 + 10\pi r^2$

$= 12\pi r^2$

$\frac{dSA}{dt} = 24\pi r \frac{dr}{dt}$

$= 24\pi r \left(\frac{2}{10\pi}\right)$

$= \frac{24r}{5}$ cm^2/sec.

Q19.

$\frac{dV}{dt} = 1$ cm^3/s.

Need $\frac{dr}{dt}$

$V = \pi r^2 h$

$= 0.02\pi r^2$

$\frac{dV}{dt} = 0.04\pi r \frac{dr}{dt}$

$= \frac{\pi r}{25} \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{25}{\pi r} \frac{dV}{dt}$

$= \frac{25}{\pi r}$.

a) $\left.\frac{dr}{dt}\right|_{r=5} = \frac{25}{\pi(r)}$

$= \frac{5}{\pi}$

$= 1.6$ cm/sec.

b) $\left.\frac{dr}{dt}\right|_{r=10} = \frac{25}{10\pi}$

$= \frac{5}{2\pi}$

$= 0.8$ cm/sec

Q20. $V = 2x^2 - 3$

a) $\frac{dV}{dt} = 4x \frac{dx}{dt}$

$a = 4x(2x^2 - 3)$

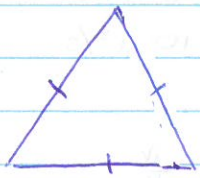
b) $v(2) = 8 - 3$

$= 5$ m/s

$a(2) = 4(2)(5)$

$= 40$ m/s^2

Q21.



$A = \frac{1}{2} l^2 \sin(60°)$

$A = \frac{\sqrt{3}}{4} l^2$

Know $l = 20$

$\frac{dl}{dt} = 0.2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2} l \frac{dl}{dt}$

$= \frac{\sqrt{3}}{2}(20)\cdot(0.2)$

$= 10\sqrt{3}\left(\frac{2}{10}\right)$

$= 2\sqrt{3}$ cm^2/sec

Q22.

$\frac{dV}{dt} = 0.5$ m^3/s

$V = \frac{4}{3}\pi r^3$.

a) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$

i) $\left.\frac{dr}{dt}\right|_{r=1} = \frac{1}{4\pi}(0.5)$

$= \frac{1}{8\pi}$

≈ 0.0398 m/sec

$\therefore \approx 4$cm/sec

ii) $\left.\frac{dr}{dt}\right|_{r=2} = \frac{1}{16\pi}(0.5)$

$= \frac{1}{32\pi}$

≈ 0.0099

$\therefore \approx 1$ cm/sec

b) After 20 seconds,

$\frac{dV}{dt} \times 20 \Rightarrow V = 0.5 \times 20$

$= 10\ m^3$.

$\frac{dr}{dt} \times 20 = \frac{20}{4\pi r^2}\frac{dV}{dt}$

$= \frac{10}{4\pi r^2}$

$\frac{dr}{dt} = \frac{1}{8\pi r^2}$

but when $V = 10$, $10 = \frac{4}{3}\pi r^3$

$\frac{30}{4\pi} = r^3$.

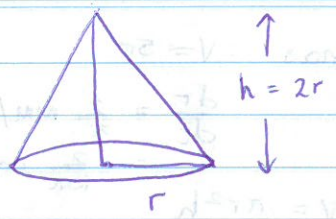
$r = \sqrt[3]{\frac{30}{4\pi}}$.

$\therefore \frac{dr}{dt} = \frac{1}{8\pi\left(\sqrt[3]{\frac{30}{4\pi}}\right)^2}$

$= 0.0223$ m/s

≈ 22 mm/s

Q23.



$\frac{dV}{dt} = 0.25\ m^3/min$

a) Need $\left.\frac{dr}{dt}\right|_{r=2}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (2r)$

$V = \frac{2}{3}\pi r^3$

$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{2\pi r^2}\frac{dV}{dt}$

$\left.\frac{dr}{dt}\right|_{r=2} = \frac{1}{8\pi}\left(\frac{1}{4}\right)$

$= \frac{1}{32\pi}$ m/min

b) Need $\left.\frac{dh}{dt}\right|_{h=2r}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

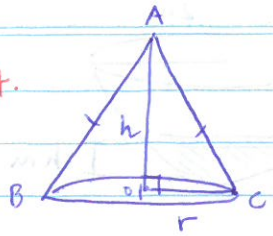
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}$

$\left.\frac{dh}{dt}\right|_{h=2} = \frac{1}{\pi}\left(\frac{1}{4}\right)$

$= \frac{1}{4\pi}$ m/min

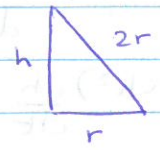
Q24.



$\frac{dV}{dt} = V \text{ cm}^3/\text{sec}$

$\left.\frac{dr}{dt}\right|_{r=20} = 0.5 \text{ cm/sec.}$

$V = \frac{1}{3}\pi r^2 h$



$h = \sqrt{4r^2 - r^2}$

$h = \sqrt{3r^2}$

$\therefore V = \frac{1}{3}\pi r^2 \sqrt{3r^2}$

$= \frac{\sqrt{3}}{3}\pi r^3.$

$\frac{dV}{dt} = \sqrt{3}\pi r^2 \frac{dr}{dt}$

$= \sqrt{3}\pi (20)^2 (0.5)$

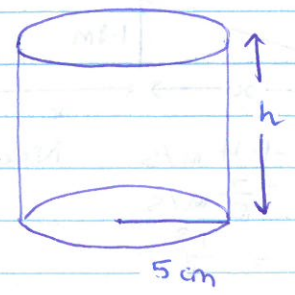
$= \frac{\sqrt{3}}{2}\pi (400)$

$= 200\sqrt{3}\pi$

$= 1088.28$

$\therefore \approx 1090 \text{ cm}^3/\text{sec (3.s.f.)}$

Q25.



$\frac{dh}{dt} = 0.1 \text{ cm/sec.}$

a) $SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(25) + 10\pi h$

$= 50\pi + 10\pi h$

$\frac{dSA}{dt} = 10\pi \frac{dh}{dt}$

$= 10\pi \left(\frac{1}{10}\right)$

$= \pi \text{ cm}^2/\text{sec}$

(*) Independent of time.

b) $V = \pi r^2 h$

$V = 25\pi h$

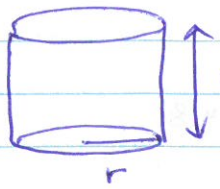
$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$= 25\pi \left(\frac{1}{10}\right)$

$= \frac{5\pi}{2} \text{ cm}^3/\text{sec}$

$= \frac{5\pi}{2} \text{ mL/sec}$

Q26.



$\frac{dr}{dt} = 0.1 \text{ cm/sec}$

a) $SA = 2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + 20\pi r$

$\frac{dSA}{dt} = 4\pi r \frac{dr}{dt} + 20\pi \frac{dr}{dt}$

$= (4\pi r + 20\pi) \frac{dr}{dr}$

$= \frac{4\pi r + 20\pi}{10}$

after 20 seconds, $r = 5 + 20(0.1)$

$= 7 \text{ cm}$

$= \frac{48\pi}{10} \text{ cm}^2/\text{sec}$

b) $V = \pi r^2 h$

$= 10\pi r^2$

$\frac{dV}{dt} = 20\pi r \frac{dr}{dt}$

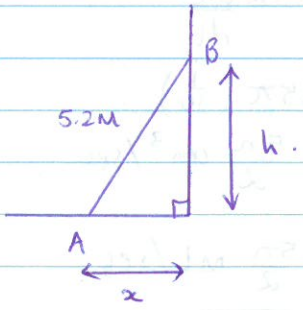
$= \frac{20\pi r}{10}$

$= 2\pi r$

after 20 seconds, $r = 7$ cm

$\therefore \frac{dV}{dt} = 14\pi \text{ cm}^3/\text{sec}$

Q27.



$\frac{dx}{dt} = 0.1 \text{ m/s}.$

Need $\left.\frac{dh}{dt}\right|_{h=4.8}$.

$x^2 + h^2 = 5.2^2$

$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 0.$

$\frac{x}{5} + 2h\frac{dh}{dt} = 0$

$2h\frac{dh}{dt} = -\frac{x}{5}$

$\frac{dh}{dt} = \frac{-x}{10h}$

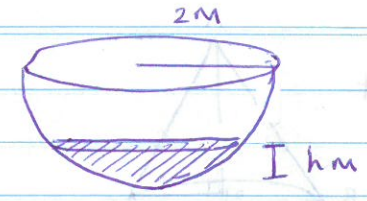
when $h = 4.8$, $x^2 = 5.2^2 - 4.8^2$

$x = 2$

$\therefore \frac{dh}{dt} = \frac{-2}{48}$

$\therefore$ decreasing by $\frac{1}{24}$ m/s.

Q28



$V = \frac{\pi}{3}h^2(6-h)$

$= 2\pi h^2 - \frac{\pi}{3}h^3.$

$\frac{dV}{dt} = -0.25 \text{ M}^3/\text{MIN}$

Need $\left.\frac{dh}{dt}\right|_{h=1}$ in MM/MIN

$\frac{dV}{dt} = 4\pi h\frac{dh}{dt} - \pi h^2\frac{dh}{dt}$

$-0.25 = (4\pi h - \pi h^2)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-1}{4(4\pi h - \pi h^2)}$

$\left.\frac{dh}{dt}\right|_{h=1} = \frac{-1}{4(4\pi - \pi)}$

$= -\frac{1}{12\pi}$ m/MIN.

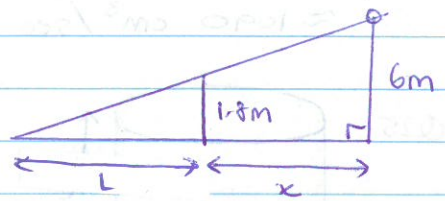
(X 1000)

$= -\frac{1000}{12\pi}$

$= -26.53$

$\therefore \approx 27$ MM/MIN decrease

Q29.



$\frac{dx}{dt} = -1.4$ M/S Need $\frac{dL}{dt}$.

$\frac{dx}{dt} = -\frac{14}{10}$ M/S.

$\frac{L}{L+x} = \frac{1.8}{6}$

$L = \frac{3}{10}(L+x)$

a) $\frac{dL}{dt} = \frac{3}{10}\left(\frac{dL}{dt} + \frac{dx}{dt}\right)$ [⊛ could be rearranged]

$$\frac{dl}{dt} - \frac{3}{10}\frac{dl}{dt} = \frac{3}{10}\frac{dx}{dt}$$

$$\frac{7}{10}\frac{dl}{dt} = \frac{3}{10}\left(\frac{-14}{10}\right)$$

$$\frac{dl}{dt} = \frac{10}{7}\left(\frac{3}{10}\right)\left(\frac{-14}{10}\right)$$

$$= \frac{-6}{10}$$

$$= -0.6$$

∴ decreasing by 0.6 m/s.

b) Need $\frac{d}{dt}(l+x)$

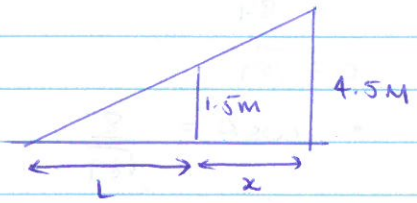
$$= \frac{dl}{dt} + \frac{dx}{dt}$$

$$= -0.6 - 1.4$$

$$= -2 \text{ m/s}$$

∴ 2 m/s.

Q30.



$\frac{dx}{dt} = 2$ m/s Need $\frac{dL}{dt}$

a) $\frac{L}{L+x} = \frac{1.5}{4.5}$

$L = \frac{1}{3}(L+x)$

$\frac{2}{3}L = \frac{1}{3}x$

$2L = x$

$2\frac{dL}{dt} = \frac{dx}{dt}$

$\frac{dL}{dt} = \frac{1}{2}\frac{dx}{dt}$

$= 1$ m/s

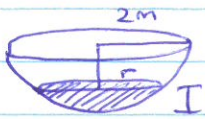
∴ increasing by 1 m/s.

b) $\frac{d}{dt}(L+x) = \frac{dL}{dt} + \frac{dx}{dt}$

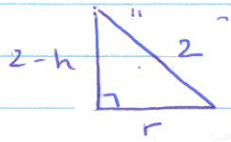
$= 1 + 2$

$= 3$ m/s

Q31.



$\frac{dh}{dt} = -0.5$ Need $\left.\frac{dr}{dt}\right|_{h=1}$.



$(2-h)^2 + r^2 = 4$

$-2(2-h)\frac{dh}{dt} + 2r\frac{dr}{dt} = 0$

$-2(2-1)\left(-\frac{1}{2}\right) + 2r\frac{dr}{dt} = 0$

$2r\frac{dr}{dt} = -1$

$\frac{dr}{dt} = \frac{-1}{2r}$

When $h=1$, $1^2 + r^2 = 2^2$

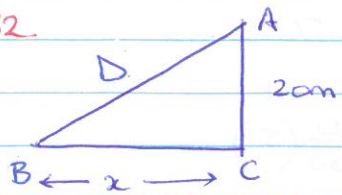
$r^2 = 3$.

$r = \sqrt{3}$.

$\frac{dr}{dt} = -\frac{1}{2\sqrt{3}}$ cm/s.

∴ decreasing by $\frac{1}{2\sqrt{3}}$ cm/sec

Q32.



$\frac{dx}{dt} = 15$ m/s

Need $\left.\frac{dD}{dt}\right|_{x=48}$

$D^2 = x^2 + 20^2$

$2D\frac{dD}{dt} = 2x\frac{dx}{dt}$

$$D\frac{dD}{dt} = x\frac{dx}{dt}$$

$$\frac{dD}{dt} = \frac{x}{D}(15)$$

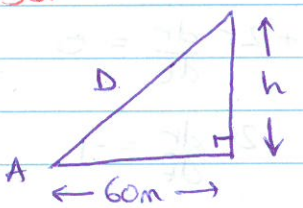
When $x = 48$

$$D^2 = 48^2 + 20^2$$
$$= 2704$$
$$D = 52$$

$$\therefore \frac{dD}{dt} = \frac{48}{52}(15)$$
$$= 13.85$$

∴ Increasing by 13.85 m/s.

Q33.



$$\frac{dh}{dt} = 5 \text{ m/s}.$$

Need $\left.\frac{dD}{dt}\right|_{h=80}$

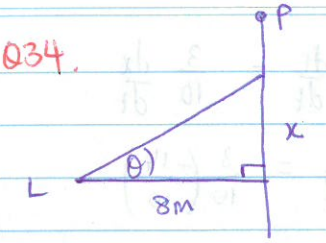
$$D^2 = 60^2 + h^2$$
$$2D\frac{dD}{dt} = 2h\frac{dh}{dt}$$
$$\frac{dD}{dt} = \frac{h}{D}\frac{dh}{dt}$$

When $h = 80$,

$$D^2 = 60^2 + 80^2$$
$$D = 100.$$
$$\therefore \frac{dD}{dt} = \frac{80}{100}(5)$$
$$= 4 \text{ M/s}$$

∴ Increasing by 4 M/s.

Q34.

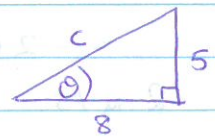


$$\frac{d\theta}{dt} = 4\pi \text{ rad/sec}$$

Need $\left.\frac{dx}{dt}\right|_{x=5}$

$$\tan\theta = \frac{x}{8}$$
$$x = 8\tan\theta$$
$$\frac{dx}{dt} = 8\sec^2\theta\frac{d\theta}{dt}$$
$$= 8\sec^2\theta(4\pi)$$
$$= 32\pi\sec^2\theta$$

When $x = 5$,



$$c^2 = 8^2 + 5^2$$
$$= 89$$
$$c = \sqrt{89}$$
$$\therefore \cos\theta = \frac{8}{\sqrt{89}}$$
$$\therefore \sec\theta = \frac{\sqrt{89}}{8}$$
$$\therefore \sec^2\theta = \frac{89}{64}$$

$$\therefore \frac{dx}{dt} = 32\pi\left(\frac{89}{64}\right)$$
$$= \frac{89\pi}{2}$$

∴ Increasing by $\frac{89\pi}{2}$ M/s

EXERCISE 8D

Q1. $f(x) = x^3 - 5x$

$f(5.01) = 5.01^3 - 5(5.01)$

$= 100.701501$

$f(5) = 5^3 - 5(5)$

$= 100$

$\therefore f(5.01) - f(5) = 0.701501$

Let $y = f(x)$

$\delta y \approx \frac{dy}{dx} \delta x$

$\delta y \approx (3x^2 - 5)(0.01)$

$\approx \frac{(3(5)^2 - 5)}{100}$

≈ 0.7

$\delta y <$ actual change by 0.001501.

Q2 $f(x) = \sin 3x$

$f(\frac{\pi}{9} + 0.01) = 0.8806$

$f(\frac{\pi}{9}) = 0.8660$

$f(\frac{\pi}{9} + 0.01) - f(\frac{\pi}{9}) = 0.01461$ ...

Let $y = f(x)$.

$\delta y \approx \frac{dy}{dx} \delta x$

$\delta y \approx 3\cos 3x (0.01)$

$\approx \frac{3\cos(\frac{\pi}{3})}{100}$

≈ 0.015

$\delta y >$ actual change by 0.00039.

Q3 $f(x) = 2\sin^3(5x)$

$f(\frac{\pi}{3} + 0.01) = -1.1852$

$f(\frac{\pi}{3}) = -1.2990$

$f(\frac{\pi}{3} + 0.01) - f(\frac{\pi}{3}) = 0.1138$

Let $y = f(x)$

$\delta y \approx \frac{dy}{dx} \delta x$

$\approx 6\sin^2(5x) \cdot 5\cos(5x)(0.01)$

$\approx 30\sin^2(5x)\cos(5x)(0.01)$

$\approx \frac{11.25}{100}$

≈ 0.1125

$\therefore \delta y <$ actual change by 0.0013.

Q4. $C = 5000 + 20\sqrt{x}$

$\frac{dC}{dx} = \frac{10}{\sqrt{x}}$

a) $\frac{dC}{dx} = \frac{10}{5}$

$= 2$

$\therefore$ \$2 / unit

b) $\frac{dC}{dx} = \frac{10}{10}$

$= 1$

$\therefore$ \$1 / unit

c) $\frac{dC}{dx} = \frac{10}{20}$

$= 0.5$

$\therefore$ \$0.50 / unit

Q5 $C = 15000 + 750x - 15x^2 + \frac{x^3}{10}$

$\frac{dC}{dx} = 750 - 30x + \frac{3x^2}{10}$

a) $\frac{dC}{dx} = 750 - 30(30) + \frac{3(900)}{10}$

$= 750 - 900 + 270$

$= 120$

$\therefore$ \$120/tonne

b) $\frac{dC}{dx} = 750 - 30(60) + \frac{3(3600)}{10}$

$= 750 - 1800 + 1080$

$= 30$

$\therefore$ \$30/tonne

c) $\frac{dC}{dx} = 750 - 30(100) + \frac{3(10000)}{10}$

$= 750 - 3000 + 3000$

$= 750$

$\therefore$ \$750/tonne

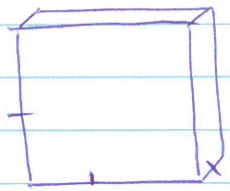
Q6. $C = 450 + 0.5x^2$

$\frac{dC}{dx} = x$

$\frac{dC}{dx}\Big|_{x=10} = 10$

$\therefore$ To produce the 11th item, it costs another \$10 //

Q7.



a) $SA = 6L^2$

$\frac{dSA}{dL} = 12L$

$\delta SA \approx 12L\,\delta L$

$\approx 60(0.2)$

$\approx 12\ cm^2$

b) $V = L^3$

$\frac{dV}{dL} = 3L^2$

$\delta V \approx 3L^2\,\delta L$

$\approx 75(0.2)$

$\approx 15\ cm^3$

EXERCISE 8E

Q1. $y = x^3(2x+1)^5$

By product rule:

$\frac{dy}{dx} = 3x^2(2x+1)^5 + x^3(5(2x+1)^4 \cdot 2)$

$= 3x^2(2x+1)^5 + 10x^3(2x+1)^4$

$= (2x+1)^4[3x^2(2x+1) + 10x^3]$

$= (2x+1)^4[6x^3 + 3x^2 + 10x^3]$

$= (2x+1)^4(16x^3 + 3x^2)$

$= x^2(16x+3)(2x+1)^4$

By log. diff:

$\ln y = \ln(x^3) + \ln[(2x+1)^5]$

$\frac{1}{y}\frac{dy}{dx} = \frac{3x^2}{x^3} + \frac{5(2x+1)^4 \cdot 2}{(2x+1)^5}$

$\frac{dy}{dx} = y\left[\frac{3x^2(2x+1)^5 + 10x^3(2x+1)^4}{x^3(2x+1)^5}\right]$

$= 3x^2(2x+1)^5 + 10x^3(2x+1)^4$

... as above

Q2. $y = \frac{x^3}{x^2+1}$

By quotient rule:

$\frac{dy}{dx} = \frac{(x^2+1)(3x^2) - (2x)(x^3)}{(x^2+1)^2}$

$= \frac{3x^4 + 3x^2 - 2x^4}{(x^2+1)^2}$

$= \frac{x^4 + 3x^2}{(x^2+1)^2}$

$= \frac{x^2(x^2+3)}{(x^2+1)^2}$

By log diff: $\ln y = \ln(x^3) - \ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{3x^2}{x^3} - \frac{2x}{x^2+1}$

$\frac{dy}{dx} = y\left(\frac{3x^2(x^2+1) - 2x^4}{x^3(x^2+1)}\right)$

... as above //

Q3 a) Let $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + 1 \ln x$

$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$

$\frac{dy}{dx} = x^x(1 + \ln x)$

b) Let $y = x^{2x}$

$\ln y = \ln(x^{2x})$

$\ln y = 2x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 2x\left(\frac{1}{x}\right) + 2\ln x$

$\frac{dy}{dx} = x^{2x}(2 + 2\ln x)$

$= 2x^{2x}(1 + \ln x)$

c) Let $y = x^{\cos x}$

$\ln y = \ln x^{\cos x}$

$\ln y = \cos x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \cos x\left(\frac{1}{x}\right) - \sin x \ln x$

$\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$

$= x^{\cos x - 1}\cos x - \sin x \ln x\, x^{\cos x}$

d) Let $y = \sqrt{\frac{3x+1}{3x-1}}$

$\ln y = \frac{1}{2}\ln\left(\frac{3x+1}{3x-1}\right)$

$\ln y = \frac{1}{2}[\ln(3x+1) - \ln(3x-1)]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{3}{3x+1} - \frac{3}{3x-1}\right]$

$\frac{1}{y}\frac{dy}{dx} = \frac{3(3x-1) - 3(3x+1)}{2(3x+1)(3x-1)}$

$\frac{dy}{dx} = y\left(\frac{9x - 3 - 9x - 3}{2(9x^2-1)}\right)$

$= \sqrt{\frac{3x+1}{3x-1}}\left(\frac{-6}{2(9x^2-1)}\right)$

$= \frac{-3}{9x^2-1}\sqrt{\frac{3x+1}{3x-1}}$

or $= \frac{-3}{(3x+1)^{\frac{1}{2}}(3x-1)^{\frac{3}{2}}}$

$= \frac{-3}{\sqrt{(3x+1)(3x-1)^3}}$